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MATHEMATICS
(LOGIC AND FOUNDATIONS)

On Formalized Systems of Algorithmic Logic

by

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Summary. The paper contains a syntactical characterization of the set of tautologies of the logical calculi based on formalized algorithmic languages. Two proofs of the completeness theorem are given.

This paper is concerned with deductive systems and theories based on formalized algorithmic languages. The languages discussed here are certain extensions of those introduced in [4]. Our task is to give a syntactical characterization of the set of tautologies. This problem was discussed in [5] for simpler languages. Here this question is made out fully in two different ways: there are given the standard and Gentzen-style axiomatizations. The theorem on the existence of models for consistent theories is also obtained.

We assume that the reader is familiar with [4]. The terminology and the notation will be here the same as in [4].

1. Languages and their realizations

Let us consider an algorithmic language

$$\mathcal{L} = \langle A, T, F, S, FS, FST, FSF \rangle$$

which is an extension of a language introduced in [4]. Sets A, T, F, S, FS (i.e. the alphabet, the set of classical terms, the set of open classical formulas, the set of substitutions and the set of programs, i.e. of FS-expressions) are defined as in [4]; the definitions of the remaining sets are given below.

The least set FST of finite sequences of signs of the alphabet A such that

t_1 $T \subset FST$,

t_2 if $\tau_1, \ldots, \tau_n$ are in FST and φ is an n-argument functor then $\varphi(\tau_1, \ldots, \tau_n)$ is in FST,

t_3 if $K \in FS$ and $\tau \in FST$ then $K\tau \in FST$,

will be called the set of terms in $\mathcal{L}$.

The least set FSF of all finite sequences of signs of A such that:

f_1 $F \subset FSF$,

f_2 if $\tau_1, \ldots, \tau_n$ are in FST and ρ is an n-argument predicate, then $\rho(\tau_1, \ldots, \tau_n)$ belongs to FSF,

f_3 if α and β are in FSF then $(\alpha \cup \beta)$, $(\alpha \cap \beta)$, $(\alpha \Rightarrow \beta)$, $\neg \alpha$ also belong to FSF,

f_4 if $K \in FS$ and α belongs to FSF, then $(K\alpha)$, $\bigcup K\alpha$, $\bigcap K\alpha$ are in FSF,

will be called the set of formulas in $\mathcal{L}$.

Let R be a realization of the language $\mathcal{L}$ in a nonempty set J and in the two-element Boolean algebra $\langle B, \wedge, \wedge, \rightarrow, - \rangle$, $B = (0, 1)$, and v — a valuation, $v \in W$, see [4]. For any $\tau \in FST$, $K \in FS$ and $\alpha \in FSF$, the realization R induces mappings τ_R, K_R, α_R. The inductive definition of partial function K_R was described in [4]. Consequently we formulate here the definitions of τ_R and α_R.

τ_R is a partial function which is defined on the set of all valuations W and takes values in the set J, namely

t_{1R} if x is an individual variable then $x_R(v) = v_x$,

t_{2R} if $\tau_1, \ldots, \tau_n$ are in FST then

$$\varphi(\tau_1, \ldots, \tau_n)_R(v) = \begin{cases} \varphi_R\left(\tau_{1_R}(v), \ldots, \tau_{n_R}(v)\right) \text{ iff } \tau_{i_R} \text{ are defined for } i = 1, \ldots, n \\ \text{is not defined — otherwise} \end{cases}$$

t_{3R} if $K \in FS$ and $\tau \in FST$ then

$$K\tau_R(v) = \begin{cases} \tau_R\left(K_R(v)\right) \text{ iff } K_R(v) \text{ and } \tau_R\left(K_R(v)\right) \text{ are defined} \\ \text{is not defined — otherwise} \end{cases}$$

By α_R we mean function

$$\alpha_R : W \rightarrow B$$

defined by induction as follows:

f_{1R} if x is a propositional variable then $x_R(v) = v_x$,

f_{2R}

$$\rho(\tau_1, \ldots, \tau_n)_R(v) = \begin{cases} \rho_R(\tau_{1_R}(v), \ldots, \tau_{n_R}(v)) \text{ iff all } \tau_{i_R}(v) \text{ are defined for } i = 1, \ldots, n \\ \quad 0 \qquad \text{otherwise} \end{cases}$$

f_{3R}

$$(\alpha \cup \beta)_R(v) = \alpha_R(v) \wedge \beta_R(v)$$
$$(\alpha \cap \beta)_R(v) = \alpha_R(v) \wedge \beta_R(v)$$
$$(\alpha \Rightarrow \beta)_R(v) = \alpha_R(v) \rightarrow \beta_R(v)$$
$$\neg \alpha_R(v) = -\alpha_R(v)$$

f_{4R}

$$(K\alpha)_R(v) = \begin{cases} \alpha_R\left(K_R(v)\right) \text{ iff } K_R(v) \text{ is defined} \\ \quad 0 \qquad \text{otherwise} \end{cases}$$

$$\bigcup K\alpha_R(v) = \bigcup_{i=0}^{\infty} (K^i \alpha)_R(v)$$

$$\bigcap K\alpha_R(v) = \bigcap_{i=0}^{\infty} (K^i \alpha)_R(v)$$

where $K^i{}_R(v)$ is defined as follows: $K_R^0(v) = v$, $K_R^i(v) = K_R^{i-1}\left(K_R(v)\right)$.

If p is an n-argument predicate and $\tau_1, ..., \tau_n$ are any elements of FST, then an expression of the form

$$p(\tau_1, ..., \tau_n)$$

is called a primitive formula.

The following two lemmas can be considered as a kind of the normal form theorems for the FST terms and for primitive formulae.

LEMMA 1. *For every n-argument functor ψ, for any terms $\tau, \tau_1, ..., \tau_{n-1}$ and any program K there exist the program K′ and term τ' such that for every realization R of the language $\mathcal{L}$ and every valuation v the following equality holds:*

$$\psi(\tau_1, ..., \tau_{i-1}, K\tau, \tau_i, ..., \tau_{n-1})_R(v) = (K' \psi(\tau_1, ..., \tau_{i-1}, \tau', \tau_i, ..., \tau_{n-1}))_R(v).$$

The proof is omitted.

LEMMA 2. *For every term $\tau \in (FST-T)$ and for every formula a of the form $p(\tau_1, ..., \tau_n)$ there exist programs K_1, K_2 and terms $\tau^*, \tau\ , ..., \tau_n^* \in T$ such that the following equalities*

(i) $\tau_R(v) = (K_1 \tau^*)_R(v)$

(ii) $a_R(v) = (K_2\, p(\tau_1^*, ..., \tau_n^*))_R(v)$

hold for all realizations R of the language $\mathcal{L}$ and for all valuations v.

This follows immediately from Lemma 1.

By assumption the set of all variables is denumerable. In the sequel we shall consider this set together with the fixed ordering r. Then by a slight modification of the proof of the Lemma 2 we get the following

Remark. Programs K_1, K_2 and terms $\tau^*, \tau_1^*, ..., \tau_n^*$ are assigned in an unambiguous way.

2. Axiomatization

Let $E(K)$ denote a formula mentioned in [4], which is valid iff the function K_R is total for every realization R and let for every substitution s and every term τ, $\bar{s\tau}$ be an expression defined as in [4].

By an axiom we shall understand any formula of one of the following forms: formulas (T_1)—(T_{12}) of [2], and formulas

A_0 $\quad ((sp(\tau_1^*, ..., \tau_n^*)) \Rightarrow p(\bar{s\tau_1^*}, ..., \bar{s\tau_n^*}))$

A_1 $\quad (p(\bar{s\tau_1^*}, ..., \bar{s\tau_n^*}) \Rightarrow (sp(\tau_1^*, ..., \tau_n^*)))$

A_2 $\quad (p(\tau_1, ..., \tau_n) \Rightarrow (Mp(\tau_1^*, ..., \tau_n^*)))$ | M and $\tau_1^* ... \tau_k^*$ are

A_3 $\quad ((Mp(\tau_1^*, ..., \tau_n^*)) \Rightarrow p(\tau_1, ..., \tau_n))$ | defined as in Lemma 2,

A_4 $\quad ((K(Ma)) \Rightarrow (\circ [KM]\, a))$

A_5 $\quad ((\circ [KM]\, a) \Rightarrow (K(Ma)))$

A_6 $\quad ((K(a \cup \beta)) \Rightarrow ((Ka) \cup (K\beta)))$

A_7 $\quad (((Ka) \cup (K\beta)) \Rightarrow (K(a \cup \beta)))$

A_8 $\quad ((K(a \cap \beta)) \Rightarrow ((Ka) \cap (K\beta)))$

$$A_9 \qquad (((K\alpha) \cap (K\beta)) \Rightarrow (K(\alpha \cap \beta)))$$

$$A_{10} \qquad ((K(\alpha \Rightarrow \beta)) \Rightarrow ((K\alpha) \Rightarrow (K\beta)))$$

$$A_{11} \qquad (E(K) \Rightarrow (((K\alpha) \Rightarrow (K\beta)) \Rightarrow (K(\alpha \Rightarrow \beta))))$$

$$A_{12} \qquad ((K \neg \alpha) \Rightarrow \neg (K\alpha))$$

$$A_{13} \qquad (E(K) \Rightarrow (\neg (K\alpha) \Rightarrow (K \neg \alpha)))$$

$$A_{14} \qquad ((M \bigcup K\alpha) \Rightarrow (M(\alpha \cup \bigcup K(K\alpha))))$$

$$A_{15} \qquad ((M(\alpha \cup \bigcup K(K\alpha))) \Rightarrow (M \bigcup K\alpha))$$

$$A_{16} \qquad ((M \bigcap K\alpha) \Rightarrow (M(\alpha \cap \bigcap K(K\alpha))))$$

$$A_{17} \qquad ((M(\alpha \cap \bigcap K(K\alpha))) \Rightarrow (M \cap K\alpha))$$

$$A_{18} \qquad ((K\alpha) \Rightarrow E(K))$$

$$A_{19} \qquad ((\underline{\vee}\,[\alpha KM]\,\beta) \Rightarrow ((\alpha \cap (K\beta)) \cup (\neg \alpha \cap (M\beta))))$$

$$A_{20} \qquad (((\alpha \cap (K\beta)) \cup (\neg \alpha \cap (M\beta))) \Rightarrow (\underline{\vee}\,[\alpha KM]\,\beta))$$

$$A_{21} \qquad ((*\,[\alpha KM]\,\beta) \Rightarrow ([c/\mathbf{1}] \bigcup \circ [[c/(c \cap \neg \alpha)]\,K]\,(c \cap \alpha \cap (M\beta))))$$

$$A_{22} \qquad (([c/\mathbf{1}] \bigcup \circ [[c/(c \cap \neg \alpha)]\,K]\,(c \cap \alpha \cap (M\beta))) \Rightarrow (*\,[\alpha KM]\,\beta))$$

where c does not occur in $(*\,[\alpha KM]\,\beta)$

$$A_{23} \qquad (([\alpha]\,\beta) \Rightarrow (\alpha \bigcap \beta))$$

$$A_{24} \qquad ((\alpha \bigcap \beta) \Rightarrow ([\alpha]\,\beta)).$$

We admit four rules of inference:

$$R_1 \qquad \frac{\alpha, (\alpha \Rightarrow \beta)}{\beta}$$

$$R_2 \qquad \frac{\alpha, E(K)}{(K\alpha)}$$

$$R_3 \qquad \frac{\{(\gamma \Rightarrow (M(K^i \alpha)))\}_{i \in N}}{(\gamma \Rightarrow (M \bigcap K\alpha))}$$

$$R_4 \qquad \frac{\{((M(K^i \alpha)) \Rightarrow \gamma)\}_{i \in N}}{((M \bigcup K\alpha) \Rightarrow \gamma)}.$$

In all the above formulas $\alpha, \beta, \gamma \in FSF$, $K, M \in FS$, $\tau_1^*, \ldots, \tau_n^* \in T$ and ρ is an n-argument predicate.

The consequence operation C is a mapping which to every set X ($X \subset FSF$) of formulas assigns the least set $C(X)$ satisfying a), b) and closed under the rules of inference:

a) $X \subset C(X)$,

b) all axioms are in $C(X)$.

In the sequel we shall study the deductive systems $\langle \mathcal{L}, C \rangle$ and formalized theories $\mathcal{T} = \langle \mathcal{L}, C, \mathcal{A} \rangle$ (where $\mathcal{A}$ is a fixed subset of FSF and will be called the set of specific axioms of $\mathcal{T}$). Deductive systems $\langle \mathcal{L}, C\rangle$ will be called systems of algorithmic logic (they can be identified with the theory $\langle \mathcal{L}, C, \emptyset\rangle$ with the empty set of specific axioms).

Any element a of the set $C(\mathcal{A})$ will be called a theorem of the theory $\langle \mathcal{L}, C, \mathcal{A}\rangle$.

For every realization R of $\mathcal{L}$ and any valuation $\hat{v}$ of propositional variables, a pair $(R, \hat{v})$ is said to be a model for theory $\mathcal{T}$ iff for every formula $a \in \mathcal{A}$ and for every valuation v^i of individual variables $a_R(v)=\mathbf{1}$, where $v=(\hat{v}, v^i)$.

THEOREM. *For every consistent theory* $\mathcal{T}=\langle \mathcal{L}, C, \mathcal{A}\rangle$ *there exists a model of* $\mathcal{T}$.

The algebraic proof makes use of the Rasiowa—Sikorski lemma, see [2].

At the end of this section we shall formulate conditions necessary and sufficient for any formula to be a theorem of a consistent theory.

THEOREM. *For every formula* a *in a consistent theory* $\mathcal{T}$ *the following conditions are equivalent*

1. a *is a theorem in* $\mathcal{T}$,
2. a *is valid in every, model of* $\mathcal{T}$.

This follows easily from the preceding theorem.

3. Diagrams

In this section we shall consider another axiomatization of the set of tautologies of the language $\mathcal{L}$. At first we recall some auxiliary notions.

Let Γ_1, Γ_2, denote finite sequences (empty sequence is admitted) of formulas in $\mathcal{L}$. Every expression of the form

$$\Gamma_1 \to \Gamma_2$$

will be called a sequent.

The sequent S

$$a_1, \ldots, a_n \to \beta_1, \ldots, \beta_m$$

is called indecomposable iff every formula a_i, β_j $(i=1, \ldots, n,\ j=1, \ldots, m)$ is a propositional variable or is a primitive formula (i.e. is of the form $\rho(\tau_1, \ldots, \tau_n)$, where $\tau_i \in T$).

A sequent S is said to be an axiom iff there exist indexes i and j $(1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m)$ such that a_i and β_j are identical.

By a scheme we shall understand a pair $\{S, S_0\}$ of sequents, a triple $\{S, S_0; S_1\}$ or an enumerable sequence of sequents $\{S, S_0; S_1; S_2; \ldots\}$, which will be written in the form

$$\frac{S}{S_0} \quad \text{or} \quad \frac{S}{S_0; S_1} \quad \text{or} \quad \frac{S}{\{S_i\}_{i\in N}}.$$

Sequent S is called conclusion and sequents S_0 in the first case S_0, S_1 in the second and S_0, S_1 ... in the third case—permises.

In the sequel we shall consider three groups of schemes: The first group:

1A) $$\frac{\Gamma_1, (s_1, \ldots, s_K a_0), \Gamma_2 \to \Gamma_3}{(s_1, \ldots, s_{k-1}\, \overline{s_k a_0}), \Gamma_1, \Gamma_2 \to \Gamma_3}$$

1B) $$\frac{\Gamma_1 \to \Gamma_2, (s_1, \ldots, s_k a_0), \Gamma_3}{\Gamma_1 \to (s_1, \ldots, s_{K-1}\, \overline{s_k a_0}), \Gamma_2, \Gamma_3}$$

where a_0 denotes an atomic formula, and k is a natural number, $k=0, 1, 2, \ldots$

2A) $$\frac{\Gamma_1, (s\rho(\tau_1, \ldots, \tau_n))\, \Gamma_2 \to \Gamma_3}{(s(K\rho(\tau_1^*, \ldots, \tau_n^*))), \Gamma_1, \Gamma_2 \to \Gamma_3}$$ 2B) $$\frac{\Gamma_1 \to \Gamma_2, (s\rho(\tau_1, \ldots, \tau_n)), \Gamma_3}{\Gamma_1 \to (s(K\rho(\tau_1^*, \ldots, \tau_n^*))), \Gamma_2, \Gamma_3}$$

where the formula $K\rho(\tau_1^*, \ldots, \tau_n^*)$ arose from $\rho(\tau_1, \ldots, \tau_n)$ in accordance with the algorithm described in the proof of Lemma 2.

3A) $$\frac{\Gamma_1, (s \neg a), \Gamma_2 \to \Gamma_3}{\Gamma_1, \Gamma_2 \to (sa), \Gamma_3}$$ 3B) $$\frac{\Gamma_1 \to \Gamma_2, (s \neg a), \Gamma_3}{\Gamma_1, (sa) \to \Gamma_2, \Gamma_3}$$

4A) $$\frac{\Gamma_1, (s(a \cap \beta)), \Gamma_2 \to \Gamma_3}{(sa), (s\beta), \Gamma_1, \Gamma_2 \to \Gamma_3}$$

5B) $$\frac{\Gamma_1 \to \Gamma_2, (s(a \cup \beta)), \Gamma_3}{\Gamma_1 \to (sa), (s\beta), \Gamma_2, \Gamma_3}$$

6B) $$\frac{\Gamma_1 \to \Gamma_2, (s(a \Rightarrow \beta)), \Gamma_3}{(sa), \Gamma_1 \to (s\beta), \Gamma_2, \Gamma_3}$$

7A) $$\frac{\Gamma_1, (s \bigcap Ka), \Gamma_2 \to \Gamma_3}{(s \bigcap K(Ka)), \Gamma_1, (sa), \Gamma_2 \to \Gamma_3}$$ 8B) $$\frac{\Gamma_1 \to \Gamma_2, (s \bigcup Ka), \Gamma_3}{\Gamma_1 \to (s \bigcup K(Ka)), \Gamma_2, (sa), \Gamma_3}$$

9A) $$\frac{\Gamma_1, (s(\circ [KM]\, a)), \Gamma_2 \to \Gamma_3}{(s(K(Ma))), \Gamma_1, \Gamma_2 \to \Gamma_3}$$ 9B) $$\frac{\Gamma_1 \to \Gamma_2, (s(\circ [KM]\, a)), \Gamma_3}{\Gamma_1 \to (s(K(Ma))), \Gamma_2, \Gamma_3}$$

10A) $$\frac{\Gamma_1, (s(\underline{\vee}\, [\beta KM]\, a)), \Gamma_2 \to \Gamma_3}{(s((\beta \cap (Ka)) \cup (\neg \beta \cap (Ma)))), \Gamma_1, \Gamma_2 \to \Gamma_3}$$

10B) $$\frac{\Gamma_1 \to \Gamma_2, (s(* [\beta KM]\, a)), \Gamma_3}{\Gamma_1 \to (s((\beta \cap (Ka)) \cup (\neg \beta \cap (Ma)))), \Gamma_2, \Gamma_3}$$

11A) $$\frac{\Gamma_1, (s(* [\beta KM]\, a)), \Gamma_2 \to \Gamma_3}{(s([c/1] \bigcup \circ [[c/(c \cap \neg \beta)\, K]\, (c \cap \beta \cap (Ma)))), \Gamma_1, \Gamma_2 \to \Gamma_3}$$

11B) $$\frac{\Gamma_1 \to \Gamma_2, (s(* [\beta KM]\, a)), \Gamma_3}{\Gamma_1 \to (s([c/1] \bigcup \circ [[c/(c \cap \neg \beta)]\, K]\, (c \cap \beta \cap (Ma)))), \Gamma_2, \Gamma_3}$$

where c does not occur in the formula $(* [\beta KM]\, a)$.

The second group:

4B) $$\frac{\Gamma_1 \to \Gamma_2, (s(a \cap \beta)), \Gamma_3}{\Gamma_1 \to (sa), \Gamma_2, \Gamma_3;\ \Gamma_1 \to (s\beta), \Gamma_2, \Gamma_3}$$

5A) $$\frac{\Gamma_1, (s(a \cup \beta)), \Gamma_2 \to \Gamma_3}{(sa), \Gamma_1, \Gamma_2 \to \Gamma_3;\ (s\beta), \Gamma_1, \Gamma_2 \to \Gamma_3}$$

6A) $$\frac{\Gamma_1, (s(a \Rightarrow \beta)), \Gamma_2 \to \Gamma_3}{\Gamma_1, \Gamma_2 \to (sa), \Gamma_3;\ (s\beta), \Gamma_1, \Gamma_2 \to \Gamma_3}$$

The third group:

7B) $$\frac{\Gamma_1 \to \Gamma_2, (s \bigcap Ka), \Gamma_3}{\{\Gamma_1 \to (s(K^i(a)), \Gamma_2, \Gamma_3\}_{i \in N}}$$

8A) $$\frac{\Gamma_1, (s \bigcup Ka), \Gamma_2 \to \Gamma_3}{\{(s(K^i a)), \Gamma_1, \Gamma_2 \to \Gamma_3\}_{i \in N}}$$

In all the above schemes

1) $\Gamma_1, \Gamma_2, \Gamma_3$ denote any sequents,
2) s denotes any sequence of substitutions $s_1, \ldots, s_k, k \geqslant 0$,
3) K, M — any program $K, M \in FS$.

By a tree we shall understand a set D of finite sequences of natural numbers such that if any sequence $c=(i_1, \ldots, i_n)$ is an element of D then every initial segment c_k of c, $c_k=(i_1, \ldots, i_k)$, $k \leqslant n$, is an element of D also. The empty sequence of natural numbers, denoted by $\emptyset$, belongs to every tree.

For any element $c=(i_1, \ldots, i_n)$ of the tree D, the number n is called the level of the element c in the tree D.

By the level of the tree D we shall mean the set of all elements which have the same level.

A subset D such that its elements are linearly ordered with respect to the relation "to be an initial segment" is called a branch of the tree D.

By the diagram of a formula a_0 we shall mean an ordered pair (D, d) where D is a tree and d is a mapping which to every element of the set D assigns certain non-empty sequent. The tree D and the mapping d are defined by induction of the level l of the tree D as follows:

1. if $l=0$ then the only element of this level is $\emptyset$, i.e. $\emptyset \in D$, and $d(\emptyset)$ is equal to the sequent $\to a_0$;

Suppose that we have defined all elements of the tree D with the level not higher than n.

Now we define elements of $n+1$ — level of D.

Let $c=(i_1, \ldots, i_n) \in D$ and let the sequent $d(c)$ be defined:

2. if $d(c)$ is an indecomposable sequent or an axiom, then none of the elements $c'=(i_1, \ldots, i_n, k)$, $k \in N$ belongs to D; c and $d(c)$ are called an end element and an end sequent of the tree D;
3. the sequent $d(c): S \to \nabla$ is neither indecomposable nor an axiom;

We consider two cases:

Case 1 — n is an even number

A. if the sequence ∇ contains only atomic formulae then $(i_1, \ldots, i_n, 0) \in D$ and $d(i_1, \ldots, i_n, 0)=d(c)$,

B. if a is a first on the right-hand side nonatomic formula in ∇, then we consider different forms of the formula a:

1°. if sequent $d(c)$ is a conclusion in a scheme of the group IB concerning the formula a, then $(i_1, \ldots, i_n, 0) \in D$ and $d(i_1, \ldots, i_n, 0)$ is equal to the only premise in this scheme, 2°. if the sequent $d(c)$ is a conclusion in a scheme of the group IIB, then $(i_1, \ldots, i_n, 0)$ and $(i_1, \ldots, i_n, 1)$ belongs to D and $d(i_1, \ldots, i_n, 0)$, $d(i_1, \ldots, i_n, 1)$ are the first and the second premise in this scheme,

3°. if the sequent $d(c)$ is a conclusion in a scheme of the group IIIB, then $(i_1, \ldots, i_n, k)$ are in D for any $k \in N$ and $d(i_1, \ldots, i_n, k)$ is k-th premise in this scheme,

Case 2 — n is an odd number.

Points A and B in the above definition must be changed in the following way: the sequence ∇ is replaced by Γ and groups I, II, IIB by I, II, IIA.

From this definition it immediately follows that for every formula its diagram is defined in an unambiguous way. The diagram is said to be finite iff every of its branch is finite.

THEOREM. *Formula a_0 is a tautology if and only if the diagram of formula a_0 is finite and every end sequent is an axiom.*

The proof is analogous to [3] and is omitted.

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Г. Мирковска, **О формализированных системах алгорифмической логики**

Содержание. Настоящая работа содержит синтаксическую характеристику множества тавтологий логических исчислений, опирающихся на формализированные алгорифмические языки. Проводятся два доказательства теоремы о полноте.